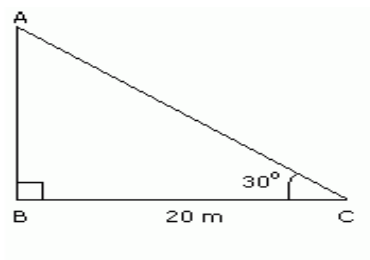


Chapter 9. Some Applications of Trigonometry

Question-1

From a point 20 m away from the foot of a tower, the angle of elevation of the top of the tower is 30° . Find the height of the tower.

Solution:



Let AB be the height of the tower and C be the point.

In rt. $\triangle ABC$,

$$\tan 30^\circ = AB/BC$$

$$AB = BC \tan 30^\circ$$

$$= \frac{20}{\sqrt{3}} = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

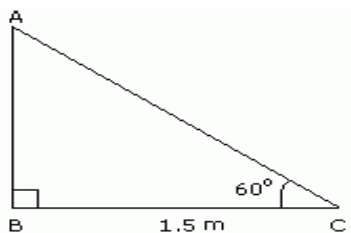
$$= 11.56 \text{ m}$$

Therefore the height of the tower is 11.56 m.

Question-2

A ladder is placed against a wall such that it just reaches the top of the wall. The foot of the ladder is 1.5 m away from the wall and the ladder is inclined at an angle of 60° with the ground. Find the height of the wall.

Solution:



Let AC be the ladder and B be the foot of the wall.

In rt. $\triangle ABC$,

$$\tan 60^\circ = AB/BC$$

$$AB = BC \tan 60^\circ$$

$$= 1.5 \sqrt{3} \text{ m}$$

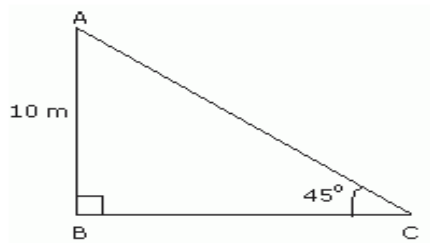
$$= 2.598 \text{ m}$$

Therefore the height of the wall is 2.598 m.

Question-3

An electric pole is 10 m high. A steel wire tied to the top of the pole is affixed at a point on the ground to keep the pole upright. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire.

Solution:



Let AB be the height of the electric pole and AC be the length of the wire.

In rt. $\triangle ABC$,

$$\operatorname{cosec} 45^\circ = AC/AB$$

$$AC = AB \operatorname{cosec} 45^\circ$$

$$= 10\sqrt{2} \text{ m}$$

$$= 10 \times 1.414 \text{ m}$$

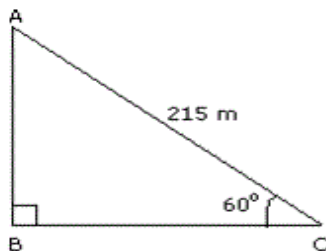
$$= 14.14 \text{ m}$$

Therefore the length of the wire is 14.14 m.

Question-4

A balloon is connected to a meteorological ground station by a cable of length 215 m inclined at 60° to the horizontal. Determine the height of the balloon from the ground. Assume that there is no slack in the cable.

Solution:



Let A be the position of the balloon.

In rt. $\triangle ABC$,

$$\sin 60^\circ = AB/AC$$

$$AB = 215 \sin 60^\circ$$

$$AB = 215 \times \frac{\sqrt{3}}{2}$$

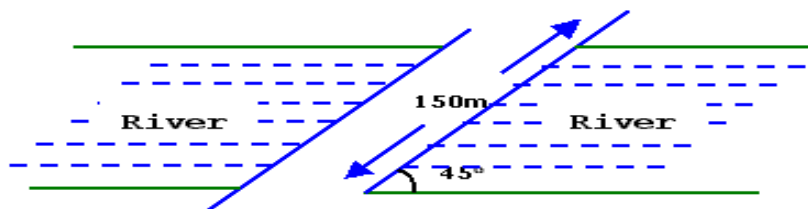
$$= 215 \times \frac{1.73}{2}$$

$$= 186 \text{ m}$$

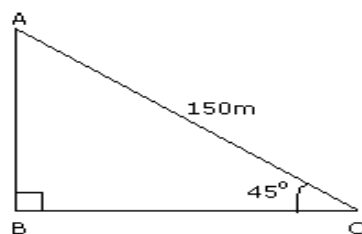
Therefore the height of the balloon from the ground is 186 m.

Question-5

A bridge across a river makes an angle of 45° with the river bank (fig.). If the length of the bridge across the river is 150 m, what is the width of the river?



Solution:



Let AB be the width of the river.

In rt. $\triangle ABC$,

$$\sin 45^\circ = AB/AC$$

$$AB = AC \sin 45^\circ$$

$$= 150 \times \frac{1}{\sqrt{2}}$$

$$= 150 \times \frac{\sqrt{2}}{2}$$

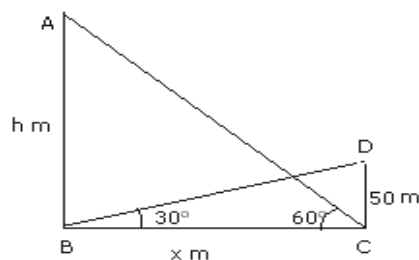
$$= 106.05 \text{ m}$$

Therefore the width of the river is 106.05 m.

Question-6

The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of top of the tower from the foot of the hill is 30° . If the tower is 50 m high, what is the height of the hill?

Solution:



Let h be the height of the hill and x m be the distance between the foot of the hill and foot of the tower.

In rt. ΔABC ,

$$\cot 60^\circ = \frac{x}{h}$$

$$x = h \cot 60^\circ \dots\dots\dots(i)$$

In rt. ΔDBC ,

$$\cot 30^\circ = \frac{x}{50}$$

$$x = 50 \cot 30^\circ \dots\dots\dots(ii)$$

Equating (i) and (ii)

$$h \cot 60^\circ = 50 \cot 30^\circ$$

$$h = 50 \cot 30^\circ / \cot 60^\circ$$

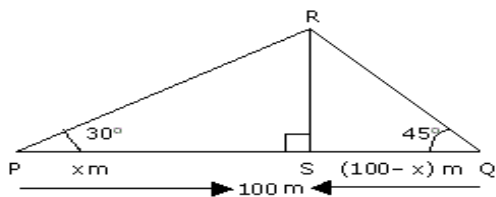
$$h = 50 \times \frac{\sqrt{3}}{1} = 50 \times 3 = 150 \text{ m}$$

Therefore the height of the hill is 150 m.

Question-7

There is a small island in the middle of a 100 m wide river and a tall tree stands on the island. P and Q are points directly opposite each other on the two banks, and in line with the tree. If the angles of elevation of the top of the tree from P and Q are respectively 30° and 45° , find the height of the tree.

Solution:



Let PQ be the width of the river and RS be the height of the tree on the island.

In rt. ΔPRS ,

$$x = RS \cot 30^\circ$$

$$x = RS \sqrt{3}$$

$$x = \sqrt{3} RS \dots\dots\dots(i)$$

In rt. ΔRSQ ,

$$SQ = RS \cot 45^\circ$$

$$(100 - x) = RS$$

$$x = 100 - RS \dots\dots\dots(ii)$$

Equating (i) and (ii) we have:

$$\sqrt{3} RS = 100 - RS$$

$$2.73 RS = 100$$

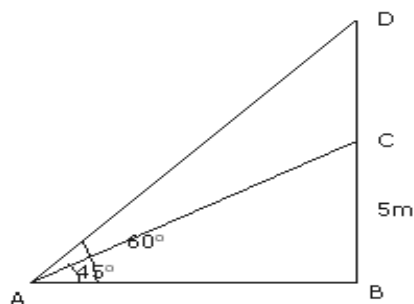
$$RS = 36.63 \text{ m}$$

Therefore the height of the tree is 36.63 m.

Question-8

A flag-staff stands at the top of a 5 m high tower. From a point on the ground, the angle of elevation of the top of the flag-staff is 60° and from the same point, the angle of elevation of the top of the tower is 45° . Find the height of the flag-staff.

Solution:



Let BC be the height of the tower and DC be the height of the flag-staff.

In rt. $\triangle ABC$,

$$AB = BC \cot 45^\circ$$

$$AB = 5 \text{ m} \dots\dots\dots(i)$$

In rt. $\triangle ABD$,

$$AB = BD \cot 60^\circ$$

$$AB = (5 + CD) \frac{1}{\sqrt{3}} \dots\dots\dots(ii)$$

Equating (i) and (ii)

$$(5 + CD) \frac{1}{\sqrt{3}} = 5$$

$$(5 + CD) = 5\sqrt{3}$$

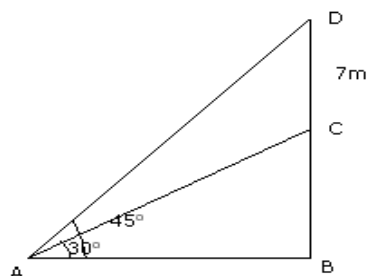
$$CD = 5\sqrt{3} - 5 = 5(1.732 - 1) = 5 \times 0.732 = 3.66 \text{ m}$$

Therefore the height of the flag-staff is 3.66 m

Question-9

A vertical tower stands on a horizontal plane and is surmounted by a flag-staff of height 7 m. From a point on the plane, the angle of elevation of the bottom of the flag-staff is 30° and that of the top of the flag-staff is 45° . Find the height of the tower.

Solution:



Let BC be the height of the tower and DC be the height of the flag-staff.

In rt. $\triangle ABC$,
 $AB = BC \cot 30^\circ$
 $AB = BC\sqrt{3}$ (i)
 In rt. $\triangle ABD$,
 $AB = BD \cot 45^\circ$
 $AB = (BC + CD) \cot 45^\circ$
 $AB = (BC + 7)$ (ii)
 Equating (i) and (ii)
 $(BC + 7) = BC\sqrt{3}$
 $BC(\sqrt{3} - 1) = 7$
 $BC = 7/0.73 = 9.58 \text{ m}$
 Therefore the height of the tower is 9.58 m.

Question-10

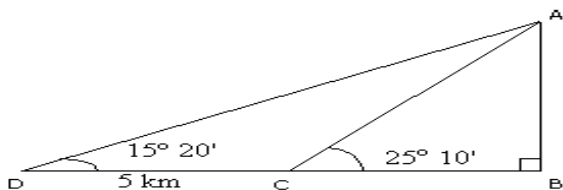
Determine the height of a mountain if the elevation of its top at an unknown distance from the base is $25^\circ 10'$ and at a distance of 5 km further off from the mountain, along the same line, the angle of elevation is $15^\circ 20'$. Give the answer in km correct to 2 decimals.

Solution:

In the figure, A is the mountain top. C and D are points of observation.

In rt. $\triangle ABC$, $\tan 25^\circ 10' = \frac{AB}{BC}$

$BC \tan 25^\circ 10' = AB$



$$BC = \frac{AB}{\tan 25^\circ 10'}$$

$$BC = \frac{AB}{0.4699}$$

In rt. $\triangle ABD$, $\tan 15^\circ 20' = \frac{AB}{BD}$

In rt. $\triangle ABD$, $\tan 15^\circ 20' = \frac{AB}{BD}$

$$\tan 15^\circ 20' = \frac{AB}{BC+5} \quad [BD = BC + 5]$$

$$0.2742 = \frac{AB}{BC+5}$$

$$\text{Substituting } BC = \frac{AB}{0.4699}$$

$$\frac{AB}{\frac{AB}{0.4699} + 5} = 0.2742$$

$$0.4699 AB = 0.2742(AB + 2.3495)$$

$$0.4699 AB - 0.2742 AB = 0.644$$

$$0.1957 AB = 0.644$$

$$AB = \frac{0.644}{0.1957} = 3.29 \text{ km}$$

$\therefore$ The height of a mountain is 3.29 km.

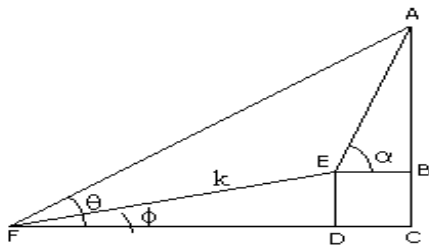
Question-11

The angle of elevation of a cliff from a fixed point A is θ . After going up a distance of k metres towards the top of the cliff at an angle ϕ , it is found that the angle of elevation is α . Show that the height of the cliff in metre is

$$\frac{k(\cos \phi - \sin \phi \cot \alpha)}{\cot \theta - \cot \alpha}.$$

Solution:

In rt. $\triangle FED$,



$$FE = k \text{ m}$$

$$\sin \phi = \frac{ED}{FE}$$

$$\therefore ED = FE \sin \phi = k \sin \phi$$

$$\text{Again, } \cos \phi = \frac{FD}{FE}$$

$$\therefore FD = FE \cos \phi = k \cos \phi \quad \dots\dots\dots(1)$$

$$\text{In rt. } \triangle AFC, \frac{FC}{AC} = \cot \theta$$

$$FC = AC \cot \theta \quad \dots\dots\dots(2)$$

$$FC - FD = DC$$

$$\text{But, } DC = EB$$

$$\therefore EB = FC - FD$$

$$\text{Thus, } EB = AC \cot \theta - k \cos \phi \quad (\text{from (1) and (2)})$$

$$\text{Now, } AB = AC - BC$$

$$\text{But, } BC = ED = k \sin \phi$$

$$\therefore AB = AC - k \sin \phi$$

$$\text{In rt. } \triangle AEB, \cot \alpha = \frac{EB}{AB}$$

$$\cot \alpha = \frac{AC \cot \theta - k \cos \phi}{AC - k \sin \phi}$$

$$AC \cot \theta - k \cos \phi = AC \cot \alpha - k \sin \phi \cot \alpha$$

$$AC \cot \theta - AC \cot \alpha = k \cos \phi - k \sin \phi \cot \alpha$$

$$AC (\cot \theta - \cot \alpha) = k \cos \phi - k \sin \phi \cot \alpha$$

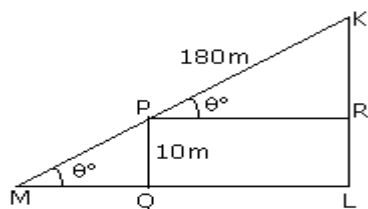
$$AC = \frac{k(\cos \phi - \sin \phi \cot \alpha)}{\cot \theta - \cot \alpha}$$

Hence proved the height of the cliff in metres is $\frac{k(\cos \phi - \sin \phi \cot \alpha)}{\cot \theta - \cot \alpha}$.

Question-12

The length of a string between a kite and a point on the roof of the building 10 m high is 180 m. If the string makes an angle θ with the level ground such that $\tan \theta = 4/3$ how high is the kite from the ground?

Solution:



In ΔKPR , $\angle KPR = \theta$, $KP = 180$ m and $\tan \theta = \frac{4}{3}$

$$\tan \theta = \frac{4}{3} = \frac{\text{opposite side}}{\text{adjacent side}} \Rightarrow \text{Hypotenuse} = \sqrt{4^2 + 3^2} = 5 \Rightarrow \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{4}{5} \Rightarrow \sin \theta = \frac{KR}{KP} = \frac{4}{5} \Rightarrow \frac{KR}{180} = \frac{4}{5} \therefore KR = \frac{4 \times 180}{5} = 144 \text{ m}$$

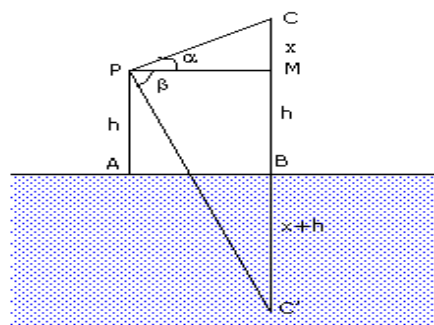
In rectangle PQLR, $PQ = RL = 10$ m (Opposite sides of a rectangle) $\therefore KL = KR + RL = (144 + 10)$ m
 $= 154$ m

$\therefore$ The height of the kite from the ground is 154 m.

Question-13

If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β , prove that the height of the cloud is $\frac{h(\tan \alpha + \tan \beta)}{(\tan \beta - \tan \alpha)}$.

Solution:



Let AB be the surface of the lake and let P be the point of the observation such that $AP = h$ metres. Let C be the position of the cloud and C' be its reflection in the lake.

Then, $CB = C'B$. Let PM be perpendicular from P on CB. Then $\angle CPM = \alpha$ and $\angle MPC' = \beta$. Let $CM = x$.

Then $CB = CM + MB = CM + MB = CM + PA = x + h$.

In ΔCPM , we have

$$\tan \alpha = CM / PM \Rightarrow \tan \alpha = x / AB \text{ [Since } PM = AB] \Rightarrow AB = x \cot \alpha \text{ -----(i)}$$

In $\Delta PMC'$, we have

$$\tan \beta = C'M / PM \Rightarrow \tan \beta = x + 2h / AB \text{ [Since } C'M = C'B + BM = x + h + h] \Rightarrow AB = (x + 2h) \cot \beta \text{ -----(ii)}$$

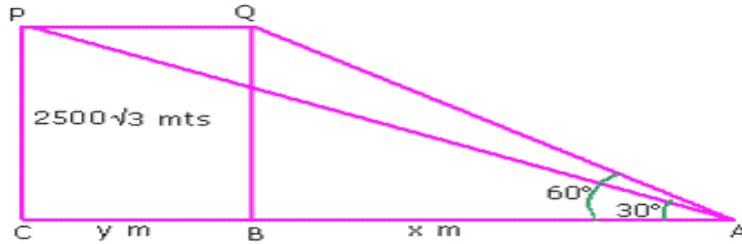
From (i) and (ii),

$$x \cot \alpha = (x + 2h) \cot \beta \Rightarrow x(\cot \alpha - \cot \beta) = 2h \cot \beta \Rightarrow x(1/\tan \alpha - 1/\tan \beta) = 2h / \tan \beta \Rightarrow x = \frac{2h \tan \alpha}{(\tan \beta - \tan \alpha)} \therefore \text{The height of the cloud} = x + h = \frac{2h \tan \alpha}{(\tan \beta - \tan \alpha)} + h = \frac{h(\tan \alpha + \tan \beta)}{(\tan \beta - \tan \alpha)}$$

Question-14

An aeroplane flying horizontally at height of $2500\sqrt{3}$ mts above that ground; is observed to be at angle of elevation 60° from the ground. After a flight of 25 seconds the angle of elevation is 30° . Find the speed of the plane in m/sec.

Solution:



Let P and Q be the two positions of the aeroplane.

$$QB = 2500\sqrt{3} \text{ mts}$$

Let the speed of the aeroplane be s m/sec.

$$\tan 60^\circ = \frac{2500\sqrt{3}}{x}$$

$$\Rightarrow \frac{2500\sqrt{3}}{x} = \sqrt{3} \therefore x = \frac{2500\sqrt{3}}{\sqrt{3}} = 2500 \text{ m}$$

$$\tan 30^\circ = \frac{2500\sqrt{3}}{x+y} \Rightarrow \frac{2500\sqrt{3}}{x+y} = \frac{1}{\sqrt{3}} \Rightarrow x+y = 2500\sqrt{3} \times \sqrt{3}$$

$$= 2500 \times 3$$

$$= 7500 \therefore y = 7500 - 2500$$

$$= 5000 \text{ m}$$

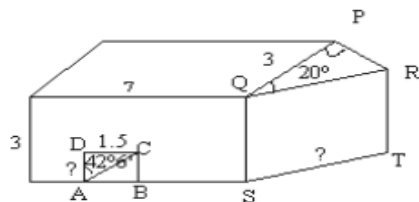
To travel 5000 m the aeroplane takes 25 seconds. $\therefore$ The speed of the plane

$$= \frac{5000}{25} = 200 \text{ m/sec.}$$

Question-15

In the figure below, some dimensions of a hut have been marked. Find the other dimensions (marked ?) of the hut correctly up to one place of decimal.

Solution:



In rt, ΔPQR , $\angle P = 90^\circ$

$$\angle PQR = 20^\circ$$

$$\sec 20^\circ = \frac{QR}{PQ}$$

$$1.0642 = \frac{QR}{3}$$

$$QR = 1.0642 \times 3 = 3.1926$$

$$QR = ST = 3.2 \text{ units}$$

In rt. $\triangle ACD$, $\angle CAD = 42^\circ 6'$

$$\cot 42^\circ 6' = \frac{AD}{DC}$$

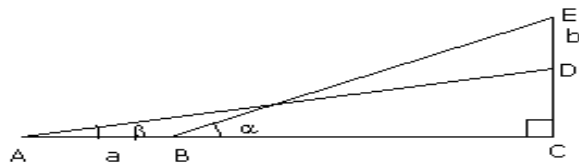
$$1.1067 = \frac{AD}{1.5}$$

$$\therefore AD = 1.5 \times 1.1067 = 1.66 \text{ units}$$

Question-16

A ladder rests against a wall at an angle α to the horizontal. When its foot is pulled away from the wall through a distance a , it slides a distance b down the wall and makes an angle β with the horizontal. Show that $a/b = (\cos \alpha - \cos \beta) / (\sin \alpha - \sin \beta)$.

Solution:



Let the length of the ladder be l units.

In rt. $\triangle ACD$,

$$\sin \beta = \frac{DC}{AD} = \frac{DC}{l}, \cos \beta = \frac{a + BC}{AD} = \frac{a + BC}{l} \dots\dots\dots(i)$$

In rt. $\triangle EBC$,

$$\sin \alpha = \frac{b + DC}{BE} = \frac{b + DC}{l}, \cos \alpha = \frac{BC}{BE} = \frac{BC}{l} \dots\dots\dots(ii)$$

From (i) and (ii)

$$\text{R.H.S} = \frac{\cos \alpha - \cos \beta}{\sin \beta - \sin \alpha} = \frac{\frac{BC}{l} - \frac{a + BC}{l}}{\frac{DC}{l} - \frac{b + DC}{l}} = \frac{BC - a - BC}{DC - b - DC} = \frac{-a}{-b} = \frac{a}{b} = \text{L.H.S.}$$

Question-17

The line joining the top of a hill to the foot of the hill makes an angle of 30° with the horizontal through the foot of the hill. There is one temple at the top of the hill and a guest house half way from the foot to the hill. The top of the temple and the top of the guesthouse both make an elevation of 32° at the foot of the hill. If the guesthouse is 1 km away from the foot of the hill along the hill, find the heights of the guest house and the temple.

Solution:

In the figure, GB is the hill. AG is the temple. EF is guesthouse. C is foot of the hill.

To find EF and AG.

$$CE = 1 \text{ km or } 1000 \text{ m}$$

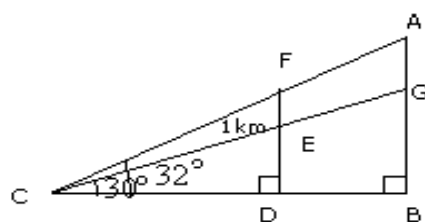
$$\text{In rt. } \triangle CED, \frac{DC}{CE} = \cos 30^\circ$$

$$\frac{CD}{1000} = \frac{\sqrt{3}}{2}$$

$$CD = 1000 \frac{\sqrt{3}}{2} = 866 \text{ m}$$

$$\frac{DE}{CE} = \sin 30^\circ$$

$$\frac{DE}{1000} = \frac{1}{2}$$



$$DE = \frac{1}{2} \times 1000 = 500 \text{ m}$$

In rt. $\triangle CDF$,

$$\frac{DF}{CD} = \tan 32^\circ$$

$$\frac{DF}{866} = 0.6249$$

$$\therefore DF = 0.6249 \times 866 = 541.16 \text{ m}$$

$$DE = 500 \text{ m.}$$

$$\therefore EF = 541.16 - 500 = 41.16 \text{ m}$$

Since E is midpoint of CG. (given halfway)

$$\therefore CG = 2000 \text{ m.}$$

$$\frac{BG}{CG} = \sin 30^\circ$$

$$\frac{BG}{2000} = \frac{1}{2}$$

$$\therefore BG = 1000 \text{ m}$$

In rt. $\triangle CBG$,

$$\frac{CB}{CG} = \cos 30^\circ$$

$$\frac{CB}{2000} = \frac{\sqrt{3}}{2}$$

$$\therefore CB = 1732 \text{ m}$$

$$\therefore CB = 1732 \text{ m}$$

In $\triangle CDF$ and $\triangle CBA$,

$$\angle CDF = \angle CBA = 90^\circ$$

$$\angle DCF = \angle BCA \text{ (common)}$$

$$\therefore \triangle CDF \sim \triangle CBA$$

$$\therefore \frac{CD}{CB} = \frac{DF}{AB}$$

$$\Rightarrow \frac{866}{1732} = \frac{541.16}{AB}$$

$$\Rightarrow AB = \frac{541.16 \times 1732}{866}$$

$$\therefore AB = 1082.32 \text{ m}$$

$$\therefore AG = 1082.32 - 1000 = 82.32 \text{ m}$$

$\therefore$ The height of guest house is 41 m and the height of temple is 82 m

Question-18

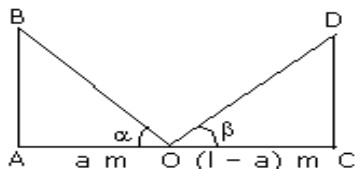
A man standing 'a' metres behind and opposite the middle of a football goal observes that the angle of elevation of the nearer cross-bar is α and that of

the further crossbar is β . Show that the length of the field is, $a(\tan\alpha \cot\beta + 1)$.

Question-18

A man standing 'a' metres behind and opposite the middle of a football goal observes that the angle of elevation of the nearer cross-bar is α and that of the further crossbar is β . Show that the length of the field is, $a(\tan\alpha \cot\beta + 1)$.

Solution:



Let AB and CD be the cross bars of the football goal.

Let O be a point, 'a' metres behind and opposite the middle of the football goal.

Let 'l' metres be the length of the field.

Let $AB = CD = p$ m since AB and CD are the cross bars of the football goal.

In rt. $\triangle BAO$,

$$\frac{AB}{AO} = \tan \alpha = \tan \alpha \cdot a = \frac{p}{\tan \alpha}$$

In rt. $\triangle DCO$,

$$\frac{CD}{CO} = \tan \beta = \tan \beta \cdot (l - a) = \frac{p}{\tan \beta}$$

Length of the field = $AO + OC$

$$\begin{aligned} &= a + (l - a) \\ &= \frac{p}{\tan \alpha} + \frac{p}{\tan \beta} \end{aligned}$$

By replacing $p = a \tan \alpha$ we get,

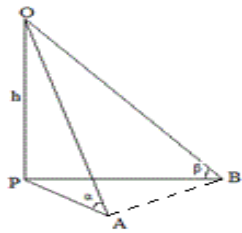
$$\begin{aligned} &= \frac{a \tan \alpha}{\tan \alpha} + \frac{a \tan \alpha}{\tan \beta} \\ &= a + \frac{a \tan \alpha}{\tan \beta} \\ &= \frac{a(\tan \beta + \tan \alpha)}{\tan \beta} \\ &= a(1 + \tan \alpha \cot \beta) \text{ m} \end{aligned}$$

Question-19

The angle of elevation of the top of a tower from a point A due south of the tower is α and from B due east of the tower is β .

If $AB = d$, show that the height of the tower is $\frac{d}{\sqrt{\cot^2 \alpha + \cot^2 \beta}}$.

Solution:



Let OP be the tower and let A and B be two points due south and east respectively of the tower such that $\angle OAP = \alpha$ and $\angle OBP = \beta$. Let $OP = h$.

In $\triangle OAP$, we have

$$\tan \alpha = \frac{h}{OA}$$

$$OA = h \cot \alpha \dots\dots\dots(i)$$

In $\triangle OBP$, we have

$$\tan \beta = \frac{h}{OB}$$

$$OB = h \cot \beta \dots\dots\dots(ii)$$

Since OAB is a right angled triangle. Therefore,

$$AB^2 = OA^2 + OB^2$$

$$d^2 = h^2 \cot^2 \alpha + h^2 \cot^2 \beta$$

$$h = \frac{d}{\sqrt{\cot^2 \alpha + \cot^2 \beta}} \text{ [Using (i) and (ii)]}.$$

Question-20

A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point b ft. just above A is β . Prove that the height of the tower is $b \tan \alpha \cot \beta$.

Solution:

Let x be the distance of the point A from the foot of the tower and h be the height of the tower.

In $\triangle APQ$,

$$h = x \tan \alpha \dots\dots(i)$$

In $\triangle PRB$

$$b = x \tan \beta \dots\dots(ii)$$

From (i) and (ii),

$$h = b \cdot \tan \alpha / \tan \beta = b \cdot \tan \alpha \cot \beta$$

Therefore the height of the tower is $b \tan \alpha \cot \beta$.

